Sector:
INFORMATION AND COMMUNICATION TECHNOLOGY

Qualification:
COMPUTER HARDWARE SERVICING NC II

Competency:
PERFORM MENSURATION AND CALCULATION

Module Title:
Performing Mensuration and Calculation

AGUSAN DEL SUR SCHOOL OF ARTS AND TRADES
D.O. Plaza Government Center, Prosperidad, Agusan del Sur
HOW TO USE THE MODULE

Welcome to the Module “Performing Mensuration and Calculation”. This module contains training materials and activities for you to complete.

The unit of competency “Perform Mensuration and Calculation” contains knowledge, skills and attitudes required for a Computer Hardware Servicing NC II course.

You are required to go through a series of learning activities in order to complete each of the learning outcomes of the module. In each learning outcome there are Information Sheets, Task Sheets and Activity Sheets. Follow these activities on your own and answer the Self-Check at the end of each learning activity.

If you have questions, don’t hesitate to ask your teacher for assistance.

Recognition of Prior Learning (RPL)

You may already have some of the knowledge and skills covered in this module because you have:

- been working for some time
- already have completed training in this area.

If you can demonstrate to your teacher that you are competent in a particular skill or skills, talk to him/her about having them formally recognized so you don’t have to do the same training again. If you have a qualification or Certificate of Competency from previous trainings show it to your teacher. If the skills you acquired are still current and relevant to this module, they may become part of the evidence you can present for RPL. If you are not sure about the currency of your skills, discuss it with your teacher.

After completing this module ask your trainer/facilitator to assess your competency. Result of your assessment will be recorded in your competency profile. All the learning activities are designed for you to complete at your own pace.

Inside this module you will find the activities for you to complete followed by relevant information sheets for each learning outcome. Each learning outcome may have more than one learning activity.
MODULE OF INSTRUCTION

Qualification : Computer Hardware Servicing NC II
Unit of Competency : Perform Mensuration and Calculation
Module Title : Performing Mensuration and Calculation
Nominal Duration : ( ) Hrs

Introduction

This module contains information and suggested learning activities on Computer Hardware Servicing II. It includes training materials and activities for you to complete.

Completion of this module will help you better understand the succeeding module on the Preparing and Interpreting Technical Drawing.

This module consists of 3 learning outcomes. Each learning outcome contains learning activities supported by each instruction sheets. Before you perform the instructions read the information sheets and answer the self-check and activities provided to as certain to yourself and your trainer that you have acquired the knowledge necessary to perform the skill portion of the particular learning outcome.

Upon completion of this module, report to your trainer for assessment to check your achievement of knowledge and skills requirement of this module. If you pass the assessment, you will be given a certificate of completion.

Learning Outcomes:
Upon completion of this module, the trainee/trainee must be able to:

1. Select measuring instrument
2. Carry out measurements and calculations
3. Maintain measuring instruments
Introduction

This Unit is concerned with measuring, calculating and estimating lengths, areas and volumes, as well as the construction of three-dimensional (3D) objects.

What is Mensuration?

In the broadest sense, mensuration is all about the process of measurement. Mensuration is based on the use of algebraic equations and geometric calculations to provide measurement data regarding the width, depth and volume of a given object or group of objects. While the measurement results obtained by the use of mensuration are estimates rather than actual physical measurements, the calculations are usually considered very accurate.

Mensuration is often based on making use of a model or base object that serves as the standard for making the calculations. From that point, advanced mathematics is employed to project measurements of length, width, and weight associated with like items. The end result is data that can help to make the best use of resources available today while still planning responsibly for the future.

Units and Measuring

Different units can be used to measure the same quantities. It is important to use sensible units. Some important units are listed below.

<table>
<thead>
<tr>
<th>1 km</th>
<th>1000 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>100 cm</td>
</tr>
<tr>
<td>1 m</td>
<td>1000 mm</td>
</tr>
<tr>
<td>1 cm</td>
<td>10 mm</td>
</tr>
<tr>
<td>1 tonne</td>
<td>1000 kg</td>
</tr>
<tr>
<td>1 kg</td>
<td>1000 g</td>
</tr>
<tr>
<td>1 litre</td>
<td>1000 ml</td>
</tr>
<tr>
<td>1 m$^3$</td>
<td>1000 litres</td>
</tr>
<tr>
<td>1 cm$^3$</td>
<td>1 ml</td>
</tr>
</tbody>
</table>

Task 1.1-1

What would be the best units to use when measuring,
(a) the distance between Butuan City and Davao City,
(b) the length of a matchbox,
(c) the mass of a person,
(d) the mass of a letter,
(e) the mass of an elf truck,
(f) the volume of medicine in a spoon,
(g) the volume of water in a swimming pool?

**Solution**

(a) Use km (or miles).
(b) Use mm or cm.
(c) Use kg.
(d) Use grams.
(e) Use tonnes
(f) Use ml.
(g) Use m3.

**Task 1.1-2**

(a) How many mm are there in 3.72 m?
(b) How many cm are there in 4.23 m?
(c) How many m are there in 102.5 km?
(d) How many kg are there in 4.32 tonnes?

**Solutions**

(a)
1 m = 1000 mm
So,
3.72 m = 3.72 × 1000
= 3720 mm

(b)
1 m = 100 cm
So,
4.23 m = 4.23 × 100
= 423 cm

1 km = 1000 m
So,

\[ 102.5 \text{ km} = 102.5 \times 1000 \]
\[ = 102500 \text{ m} \]

(d)

1 ton = 1000 kg

So

\[ 4.32 \text{ km} = 4.32 \times 1000 \]
\[ = 4320 \text{ kg} \]

**Task 1.1-3**

What value does each arrow point to?

(a) Here the marks are 0.1 units apart.

So the arrow points to 12.6.

(b) Here the marks are 0.2 units apart.

So the arrow points to 11.8.

(c) Here the marks are 0.4 units apart.

So the arrow points to 6.8.
Exercises

1. Measure each line below. Give the length to the nearest mm.
   (a) ________________________________
   (b) ________________________________
   (c) ________________________________
   (d) ________________________________
   (e) ________________________________

2. Which units do you think would be the most suitable to use when measuring:
   (a) the distance between two towns,
   (b) the length of a sheet of paper,
   (c) the mass of a sheet of paper,
   (d) the mass of a sack of cement,
   (e) the volume of a water in a cup,
   (f) the volume of water in a large tank?

3. Compute
   (a) How many grams are there in 12.3 kg?
   (b) How many mm are there in 4.7 m?
   (c) How many mm are there in 16.4 cm?
   (d) How many m are there in 3.4 km?
   (e) How many cm are there in 3.7 m?
   (f) How many ml are there in 6 litres?

4. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Length in m</th>
<th>Length in cm</th>
<th>Length in mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>311</td>
<td></td>
<td>1500</td>
</tr>
<tr>
<td>374</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.62</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Read off the value shown by the arrow on each scale.

6. A jug contains 1 litre of water.
   (a) If 150 ml is poured out, how much water is left?
   (b) A glass holds 200 ml of water. How many glasses could be filled from a full jug?

7. State whether the following lengths would be best measured to the nearest m, cm or mm.
   a. Your height
   b. The height of a hill
   c. The height of a building
   d. The width of a matchstick
   e. The length of a ship
   f. The thickness of a book

8. A cuboid has sides as shown in the diagram.
   Convert the lengths of these sides to mm.

9. Each length below is given in mm. Give each length to the nearest cm.
10.

(a) What metric unit of length would you use to measure the length of a large coach?

(b) Using the unit you gave in part (a) estimate the length of a large coach.
Estimating Areas

A square with sides of 1 cm has an area of 1 cm²

Task 1.1-4

Find the area of the shaded shape

Solution.

The shape covers 11 squares, so its area is 11 cm².

Task 1.1-5

Find the area of the shaded triangle

Solution.

The triangle covers 6 full squares marked F, and 4 half squares marked H.

Area \( \frac{6}{2} \) = 8 cm²
Task 1.1-6

Estimate the area of the shape shaded in the diagram

This is a much more complicated problem as there are only 9 full squares marked $F$, but many other part squares. You need to combine part squares that approximately make a whole square.

For example,

- the squares marked * make about 1 full square;
- the squares marked $\times$ make about 1 full square;
- the squares marked + make about 1 full square;
- the squares marked $\cdot$ make about 1 full square.

Thus the total area is approximately

$$9 + 4 = 13 \text{ cm}^2$$
Exercises

1. Find the area of the following shapes.

2. By counting the number of whole squares and half squares, find the area of each of the following shapes.
3. Estimate the area of each of the following shapes.

4. The diagrams below shows the outlines of two islands, A and B. The grid squares have sides of length 1 km. Find the approximate area of each island.
5. Each of the squares in this grid has an area of 1 square centimeter. Work out the area of the shaded shape.
Making Solids Using Nets

A net can be cut out, folded and glued to make a hollow shape.

In this Unit, you will be dealing with 3-dimensional shapes such as 

- cuboid
- prism
- pyramid
- tetrahedron

Task 1.1-7

What solid is made when the net shown is folded and glued?

Solution

It is important to add tabs to the net so that it can be glued. You could put tabs on every edge, but this would mean gluing tabs to tabs. The diagram opposite shows one possible position of the tabs.

Before gluing, crease all the folds.

The final solid is a triangular prism.
Exercises

1. Copy and cut out larger versions of the following nets. Fold and glue them to obtain cubes. Do not forget to add tabs to the nets.

1. Copy each net shown below make it into a solid. State the name of the solid that you make, if it has one.
2. The diagram shows the net for a dice with some of the spots in place. Fill in the missing spots so that the opposite faces add up to 7. Then make the dice.
Constructing Nets
A net for a solid can be visualized by imagining that the shape is cut along its edges until it can be laid flat.

Task 1.1-8

Draw the net for the cuboid shown in the diagram.

Solution

Imagine making cuts as below:
• cut along the edges AB, BC and CD to open the top like a flap.
• then cut down AE, BF, CG and DH, and press flat to give the net below.
**Task 1.1-9**

Draw the net for this square based pyramid.

**Solution**

First imagine cutting down the edges AD and AC and opening out a triangle.

Then cutting down AB and AE gives the net below.
Exercises

1. Draw an accurate net for each cuboid below.

(a)  
```
  2 cm          4 cm
  2 cm
```

(b)  
```
  1 cm       4 cm
  3 cm

  4 cm
```

(c)  
```
  2 cm  4 cm
  1 cm
```

(d)  
```
  2 cm          3 cm
  2.5 cm
```

2. Draw a net for each of the following solids.

(a)  
```
  4 cm
  5 cm
  3 cm

  3 cm
```

(b)  
```
  4 cm
  4 cm
  5 cm

  3 cm
```

(c)  
```
  All edges 5 cm
```

(d)  
```
  2 cm          6 cm
  2 cm
  2 cm
  3 cm
```
3.  
(a) Draw and cut out four equally sized equilateral triangles.  
(b) How many different ways can they be arranged with sides joined together?  
   One example is shown.  
(c) Which of your arrangements of triangles form a net for a tetrahedron?

4. The diagrams below show the ends of two of prisms that each have length of 8 cm. Draw a net for each prism.

5. Which one of these nets can be folded to make a cube?
6. The diagram above shows a pyramid with four equal triangular faces. Each edge is 4 cm long. Below is one of the faces.

(a) What is the special name given to this kind of triangle?

(b) What is the size of each angle of this triangle?

(c) Construct an accurate net for the pyramid. One face has been drawn for you.

**Conversion of Units**

It is useful to be aware of both metric and imperial units and to be able to convert between them.

**Task 1.1-9**

John is measured. His height is 5 feet and 8 inches.

Find his height in:

(a) inches, (b) centimeters
(c) meters.

Solution.
(a) There are 12 inches in one foot, so
John's height  =  5 x 12 + 8
               = 60 + 8
               = 68 inches

(b) 1 inch is about 2.5 cm, so
John’s height  = 68 x 2.5
               = 170 cm

c) 1 meter = 100 cm, so
John’s height  = 1.7 m

Task 1.1-10
A family travels 365 miles on holiday. Convert this distance to km.

Solution.
As 5 miles is approximately equal to 8 km, first divide by 5 and then multiply by 8.

\[
\frac{265}{5} = 73 \\
73 \times 8 = 584
\]

As 5 miles is approximately equal to 8 km, first divide by 5 and then multiply by 8.

Task 1.1-11
Jared weighs 8 stone and 5 pounds. Find Jared's weight in:

(a) pounds,
(b) kg.

Solution
(a) There are 14 pounds in 1 stone, so
Jared's weight  = 8 x 14 + 5
112 + 5
= 117 lbs

(b) As 1 pound is about 0.45 kg,
Jared’s weight = 117 \times 0.45
= 53 kg (to the nearest kg)

**Task 1.1-12**

A line is 80 cm long. Convert this length to inches.

**Solution**

1 inch = 2.5 cm
80/2.5 = 32, so the line is about 32 inches long.
Exercises

1. Convert each quantity to the units given.

   (a) 3 inches to cm
   (b) 18 stone to pounds
   (c) 6 lbs to ounces
   (d) 6 feet 3 inches to inches
   (e) 15 kg to lbs
   (f) 3 yards to inches
   (g) 3 feet to cm
   (h) 5 gallons to litres
   (i) 120 inches to cm
   (j) 45 kg to lbs
   (k) 9 litres to pints
   (l) 45 gallons to litres
   (m) 8 litres to pints
   (n) 6 gallons to pints

2. Convert each quantity to the units given. Give answers to 1 decimal place.

   (a) 8 lbs to kg
   (b) 3 lbs to kg
   (c) 16 pints to litres
   (d) 10 cm to inches
   (e) 400 cm to feet
   (f) 80 ounces to pounds
   (g) 182 lbs to stones
   (h) 50 litres to gallons
   (i) 84 inches to feet
   (j) 52 cm to inches
   (k) 16 litres to gallons
   (l) 3 pints to litres
   (m) 6 lbs to kg
   (n) 212 cm to feet

3. A car travels on average 10 km for every liter of petrol. The car is driven from Bayugan to Butuan, a distance of 41 miles.

   (a) How far does the car travel in km?
   (b) How many liters of petrol are used?
   (c) How many gallons of petrol are used?

4. A recipe for a large cake includes the following ingredients.

   \[
   \frac{3}{4} \text{ pint} \quad \text{orange juice} \\
   3 \text{ lbs} \quad \text{flour} \\
   \frac{1}{2} \text{ lb} \quad \text{butter} \\
   2 \text{ lbs} \quad \text{mixed fruit}
   \]

   Convert these units to liters or kg, giving your answers to 2 decimal places.
5. The Krishnan family is going on holiday with their caravan. The length of their car is 12 feet 10 inches and the length of their caravan is 16 feet 8 inches. Find the total length of the car and caravan in
(a) inches,  
(b) cm,  
(c) meters.

6. James is 6 feet 2 inches tall and weighs 11 stone 5 pounds.
Michael is 180 cm tall and weighs 68 kg.
Who is the taller and who is the heavier?

7. Jane and Christopher go strawberry picking. Jane picks 8 kg and Christopher picks 15 lbs. Who has picked the greater weight of strawberries?

8. A customer asks for a sheet of glass 15 inches by 24 inches. What would be the area of the glass in cm\(^2\)?

9. Rohan is going to buy a new car. He tries out two different ones. The first car he tries out travels 50 miles on 2 gallons of petrol. The second car travels 100 km on 12 litres of petrol. Find the petrol consumption in litres per km for both cars. Which is the more economical?

10. Here is a rule to change miles into kilometers.

<table>
<thead>
<tr>
<th>Multiply the number of miles by 8</th>
<th>Divide by 5</th>
</tr>
</thead>
</table>

(a) Use this rule to change 30 miles into kilometers.
(b) Write down an equation connecting kilometers (K) and miles (M).
(c) Use your equation to find the value of M when K = 100.

11. (a) Convert 48 kg to grams.
A box contains 280 hockey balls.
The hockey balls weigh 48 kg.
(b) Calculate the weight of one
hockey ball to the nearest gram.

One kilogram is approximately 2.2 pounds.

(c) Estimate the weight of the box of hockey balls in pounds.

12. The same quantity can sometimes be measured in different units.
   (a) Write out the statement below, filling in the missing unit.
       Choose the unit from this list:
       millimetres, centimetres, metres, kilometers
       
       1 inch = 2.54 . . . . . . . . . . . . . . . . . .
   (b) Write out the statement below, filling in the missing unit.
       Choose the unit from this list:
       millimetres, litres, gallons, cubic metres
       
       4 pints = 2.27 . . . . . . . . . . . . . . . . . .

13. (a) Megan is 5 feet 3 inches tall.
       1 cm = 0.394 inches
       12 inches = 1 foot
       Calculate Megan's height in centimetres. Give your answer to an appropriate degree of accuracy.

   (b) An electronic weighing scale gives Megan's weight as 63.4792 kg.
       Give her weight correct to an appropriate degree of accuracy.

14. A ball bearing has mass 0.44 pounds
    1 kg = 2.2 pounds
    (a) Calculate the mass of the ball bearing in kilograms
    (b) Density = \( \frac{\text{Mass}}{\text{Volume}} \)

When the mass is measured in kg and the volume is measured in cm\(^3\), what are the units of the density?

15. A recipe for a cake for four people uses
    4 eggs
8 ounces sugar
14 ounces flour
¼ pint milk

16 ounces = 1 pound

James finds a 500 g bag of flour in the cupboard. Will he have enough flour for this recipe? Clearly explain your reasoning.
Squares, Rectangles and Triangles

For a *square*, the area is given by \( x \cdot x = x^2 \) and the perimeter by \( 4 \cdot x \), where \( x \) is the length of a side.

For a *rectangle*, the area is given by \( l \cdot w \) and the perimeter by \( 2(l + w) \), where \( l \) is the length and \( w \) the width.

For a *triangle*, the area is given by \( \frac{1}{2}bh \) and the perimeter by \( a + b + c \), where \( b \) is the length of the base, \( h \) the height and \( a \) and \( c \) are the lengths of the other two sides.

**Task 1.1-13**

Find the area of each triangle below.

**Solution.**

Use \( \text{Area} = \frac{1}{2}bh \) or \( \frac{1}{2} \times \text{base} \times \text{height} \).

(a) \[ \text{Area} = \frac{1}{2} \times 5 \times 4.2 = 10.5 \text{ cm}^2 \]

(b) \[ \text{Area} = \frac{1}{2} \times 6 \times 5.5 = 16.5 \text{ cm}^2 \]

**Task 1.1-14**

Find the perimeter and area of each shape below.
Solution.

(a) The perimeter is found by adding the lengths of all sides.

\[
P = 6 + 8 + 1 + 4 + 4 + 4 + 1 + 8
\]
\[
= 36 \text{ cm}
\]

To find the area, consider the shape split into a rectangle and a square.

Area = Area of rectangle + Area of square
\[
= 6 \times 8 + 4^2
\]
\[
= 48 + 16
\]
\[
= 64 \text{ cm}^2
\]

(b) Adding the lengths of the sides gives

\[
P = 10 + 7 + 8 + 2 + 2 + 5
\]
\[
= 34 \text{ cm}
\]

The area can be found by considering the shape to be a rectangle with a square removed from it.

Area of shape = Area of rectangle – Area of square
\[
= 7 \times 10 - 2^2
\]
\[
= 70 - 4
\]
\[
= 66 \text{ cm}^2
\]
Exercises

1. Find the area of each triangle.

2. Find the perimeter and area of each shape below
3. Find the area of each shape

(a) \[ \text{Area} = \frac{1}{2} \times (12 \text{ cm} \times 10 \text{ cm}) + (8 \text{ cm} \times 10 \text{ cm}) \]

(b) \[ \text{Area} = \frac{1}{2} \times (11 \text{ cm} \times 4 \text{ cm}) + (8 \text{ cm} \times 4 \text{ cm}) \]

(c) \[ \text{Area} = \frac{1}{2} \times (6 \text{ cm} \times 4 \text{ cm}) + (7 \text{ cm} \times 2 \text{ cm}) \]

(d) \[ \text{Area} = \frac{1}{2} \times (3 \text{ cm} \times 2 \text{ cm}) + (5 \text{ cm} \times 2 \text{ cm}) \]

4. The diagram shows the end wall of a shed built out of concrete bricks.

   a. Find the area of the wall
   b. The blocks are 45 cm by 23 cm in size.
5. How many blocks would be needed to build the wall? (The blocks can be cut)

4. The shaded area on the speed-time graph represents the distance travelled by a car.

Find the distance.

6. The plan shows the base of a conservatory.

Find the area of the base.

7. The diagram shows the two sails from a dinghy.

Find their combined area.

8. The diagram shows the letter V.

Find the area of this letter.

9. Find the area of the arrow shown in the diagram.
10. The diagram shows how the material required for one side of a tent is cut out.

   (a) Find the area of the material shown if: 
       \[ b = 3.2 \text{ m}, \quad c = 2 \text{ m} \]
       (i) \[ a = 1.5 \text{ m} \]  (ii) \[ a = 2 \text{ m} \]
   (b) Find the area if \[ a = 1.6 \text{ m}, \]
       \[ b = 3.4 \text{ m}, \quad \text{and} \quad c = 2 \text{ m}. \]

11. The \[ E \] shape above is shaded on centimeter squared paper

   (a) Find the perimeter of this shape
   (b) Find the area of this shape.

12. (a) What is the perimeter of the rectangle?
    (b) What is the area of the triangle?
13. Work out the areas of these shapes

(a) \[ \text{3 cm} \quad \text{6 cm} \]

(b) \[ \text{5 cm} \quad \text{12 cm} \]

14. Calculate the area of this shape

\[ \text{8 cm} \]

\[ \text{4 cm} \]

\[ \text{2 cm} \]

\[ \text{3 cm} \]

15. By making and using appropriate measurements, calculate the area of triangle ABC in square centimeters. State the measurements that you have made and show your working clearly

16.
(a) Write down the coordinates of the mid-point of AC
(b) Copy the diagram and mark and label a point D so that ABCD is a rectangle.
(c) (i) Find the perimeter of the rectangle ABCD.
    (ii) Find the area of the rectangle ABCD.
(d) The rectangle has reflective (line) symmetry. Describe another type of symmetry that it has.
Area and Circumference of Circle

The circumference of a circle can be calculated using

\[ C = 2\pi r \quad \text{or} \quad C = \pi d \]

where \( r \) is the radius and \( d \) the diameter of the circle.

The area of a circle is found using

\[ A = \pi r^2 \quad \text{or} \quad A = \frac{\pi d^2}{4} \]

**Task 1.1-15**

Find the circumference and area of a circle

**Solution.**

The circumference is found using \( C = 2\pi r \), which in this case gives

\[ C = 2\pi \times 4 \]

\[ = 25.1 \text{ cm} \quad \text{(to one decimal place)} \]

The area is found using \( A = \pi r^2 \), which gives

\[ A = \pi \times 4^2 \]

\[ = 50.3 \text{ cm}^2 \quad \text{(to one decimal place)} \]

**Task 1.1-16**

Find the radius of a circle.

(a) its circumference is 32 cm, (b) its area is 14.3 cm\(^2\).

**Solution.**

(a) Using \( C = 2\pi r \) gives

\[ 32 = 2\pi r \]

and dividing by \( 2\pi \) gives

\[ \frac{32}{2\pi} = r \]

so that

\[ r = 5.10 \text{ cm} \quad \text{(to 2 decimal places)} \]
Task 1.1-17

Find the area of the door shown in the diagram.
The top part of the door is a semicircle.

Solution.

First find the area of the rectangle.
\[
\text{Area} = 80 \times 160
\]
\[
= 12800 \text{ cm}^2
\]

Then find the area of the semicircle.
\[
\text{Area} = \frac{1}{2} \times \pi \times 40^2
\]
\[
= 2513 \text{ cm}^2
\]

Total area = 12800 + 2513
\[
= 15313 \text{ cm}^2 \text{ (to the nearest cm}^2)\]
Exercises

1. Find the circumference and area of each circle shown below.

(a) \[ \text{circumference: } 5 \text{ cm} \]

(b) \[ \text{radius: } 0.2 \text{ m} \]

(c) \[ \text{diameter: } 1.2 \text{ m} \]

(d) \[ \text{diameter: } 24 \text{ cm} \]

(e) \[ \text{radius: } 1.4 \text{ m} \]

(f) \[ \text{radius: } 20 \text{ m} \]

2. Find the radius of the circle which has:
   (a) a circumference of 42 cm,
   (b) a circumference of 18 cm,
   (c) an area of 69.4 cm²,
   (d) an area of 91.6 cm².

3. The diagram shows a running track.
   (a) Find the length of one complete circuit of the track.
   (b) Find the area enclosed by the track.

\[ \text{length: } 100 \text{ m} \]
\[ \text{radius: } 50 \text{ m} \]
4. A washer has an outer radius of 1.8 cm and an inner radius of 0.5 cm. Find the area that has been shaded in the diagram, to the nearest cm$^2$.

5. An egg, fried perfectly, can be thought of as a circle (the yolk) within a larger circle (the white).

(a) Find the area of the smaller circle that represents the surface of the yolk
(b) Find the area of the surface of the whole egg.
(c) Find the area of the surface of the white of the egg, to be nearest cm$^2$.

6. The shapes shown below were cut out of card, ready to make cones. Find the area of each shape.

7. A circular hole with diameter 5 cm is cut out of a rectangle metal plate of length 10 cm and width 7 cm. Find the area of the plate when the hole has been cut out.

8. Find the area of the wasted material if two circles of radius 4 cm are cut out of a length 10 cm and width 7 cm. Find the area of the plate when the hole has been cut out.

9. A square hole is cut in a circular piece of card to create the shape shown.

(a) Find the shaded area of the card if the radius of the circle is 5.2 cm and the sides of the square are 4.8 cm.
(b) Find the radius of the circle if the shaded area is 50 cm$^2$ and the square has sides of length 4.2 cm.
10. Four semicircles are fixed to the sides of a square as shown in the diagram, to form a design for a table top.
   (a) Find the area of the table top if the square has sides of length 1.5 m.
   (b) Find the length of the sides of the square and the total area of the table top if the area of each semicircle is 1 m².

11. The radius of a circle is 8 cm.
    Work out the area of the circle.
    (Use \( \pi = 3.14 \) or the \( \pi \) button on your calculator.)

12. A circle has a radius of 15 cm.

    Calculate the area of the circle.
    Take \( \pi \) to be 3.14 or use the \( \pi \) key on your calculator.

13. Louise does a sponsored bicycle ride.
    Each wheel of her bicycle is of radius 25 cm.
    (a) Calculate the circumference of one of the wheels
    (b) She cycles 50 km. How many revolution does a wheel make during the sponsored ride?

14. The diameter of a garden rollers is 0.4 m.

    The roller is used on a path of length 20 m.
    Calculate how many times the roller rotates when rolling the length of the path once.
    Take \( \pi \) to be 3.14 or use the \( \pi \) key on your calculator.
15. A piece of rope is 12 meters long. It is laid on the ground in a circle, as shown in the diagram.

(a) Using 3.14 as the value of \( \pi \), calculate the diameter of the circle.

(b) Explain briefly how you would check the answer to part (a) mentally.

The cross-section of the rope is a circle of radius 1.2 cm.

(c) Calculate the area of the cross-section.

16.

The diagram shows a running track.

BA and DE are parallel and straight. They are each of length 90 meters.

BCD and EFA are semicircular. They each have a diameter of length 70 meters.

(a) Calculate the perimeter of the track.

(b) Calculate the total area inside the track.
Volumes of Cubes, Cuboids, Cylinders and Prisms

The volume of a cube is given by

\[ V = a^3 \]

where \( a \) is the length of each side of the cube.

For a cuboid the volume is given by

\[ V = abc \]

where \( a, b \) and \( c \) are the lengths shown in the diagram.

The volume of a cylinder is given by

\[ V = \pi r^2 h \]

where \( r \) is the radius of the cylinder and \( h \) is its height.

The volume of a triangular prism can be expressed in two ways, as

\[ V = A l \]

where \( A \) is the area of the end and \( l \) the length of the prism, or as

\[ V = \frac{1}{2} bh l \]

where \( b \) is the base of the triangle and \( h \) is the height of the triangle.

**Task 1.1-18**

The diagram shows a lorry.

Find the volume of the load-carrying part of the lorry.

**Solution**

The load-carrying part of the lorry is represented by a cuboid, so its volume is given by

\[ V = 2 \times 2.5 \times 4 = 20 \text{ m}^3 \]
Task 1.1-19
The cylindrical body of a fire extinguisher has the dimensions shown in the diagram. Find the maximum volume of water the extinguisher could hold.

**Solution.**
The body of the extinguisher is a cylinder with radius 10 cm and height 60 cm, so its volume is given by

\[ V = \pi r^2 h = \pi \times 10^2 \times 60 \]

\[ = 18850 \text{ cm}^3 \] (to the nearest cm\(^3\))

Task 1.1-20
A 'sleeping policeman' (traffic calming road hump) is made of concrete and has the dimensions shown in the diagram. Find the volume of concrete needed to make one 'sleeping policeman'.

**Solution.**
The shape is a triangular prism with \( b = 80 \) \( h = 10 \) \( l = 300 \) given by

\[ V = \frac{1}{2} \times 80 \times 10 \times 300 \]

\[ = 120000 \text{ cm}^3. \]
Exercises

1. Find the volume of each solid shown below
   (a) 
   (b) 
   (c) 
   (d) 
   (e) 
   (f) 
   (g) 
   (h) 
   (i) 

2. (a) Find the volume of the litter bin shown in the diagram, in m$^3$ to 2 decimal places.
    (b) Find the volume of rubbish that can be put in the bin, if it must all be below the level of the hole in the side, in m$^3$ to 2 decimal places.

3. A water tank has the dimensions shown in the diagram.
   (a) Find the volume of the tank.
   (b) If the depth of water is 1.2 m, find the volume of the water.
4. A concrete pillar is a cylinder with a radius of 20 cm and a height of 2 m.
   (a) Find the volume of the pillar.

   The pillar is made of concrete, but contains 10 steel rods of length 1.8 m and diameter 1.2 cm.
   (b) Find the volume of one of the rods and the volume of steel in the pillar.
   (c) Find the volume of concrete contained in the pillar.

5. The box shown in the diagram contains chocolate.
   (a) Find the volume of the box.
   (b) If the box contains 15 cm³ of air, find the volume of the chocolate.

6. Find the volume of each prism below.

7. Each diagram below shows the cross section of a prism. Find the volume of the prism, given the length specified.
8. The diagram shows the cross section of a length of guttering. Find the maximum volume of water that a 5 m length of guttering could hold.

9. The diagram shows the cross section of a skip that is 15 m in length and is used to deliver sand to building sites. Find the volume of sand in the skip when it is filled level to the top.

10. A ramp is constructed out of concrete. Find the volume of concrete contained in the ramp.

11. The diagram shows a cargo container.
12. A garage has a rectangular concrete base 6 m long and 2.5 m wide. The base is shown in the diagram.
   (a) Calculate the area of the garage floor.
   (b) The concrete is 0.2 m thick. Calculate the volume of the concrete base.
   (c) Write 0.2 m in millimetres.

13. Tomato soup is sold in cylindrical tins.

   Each tin has a base radius of 3.5 cm and a height of 12 cm.

   (a) Calculate the volume of soup in a full tin.
   Take \( \pi \) to be 3.14 or use the \( \pi \) key on your calculator.
   (b) Mark has a full tin of tomato soup for dinner. He pours the soup into a cylindrical bowl of radius 7 cm.

   What is the depth of the soup in the bowl?
The diagrams represents a swimming pool.
The pool has vertical sides.
The pool is 8 m wide.
(a) Calculate the area of the shaded cross section.
The swimming pool is completely filled with water.
(b) Calculate the volume of water in the pool.
64 m³ leaks out of the pool.
(c) Calculate the distance by which the water level falls.

15. The diagrams shows a paint trough in the shape of a prism.
Each shaded end of the trough is a vertical trapezium.

Calculate the volume of paint which the trough can hold when it is full.

16. The diagram shows a lamp.
(a) The base of the lamp is a cuboid. Calculate the volume of the base.

(b) The top of the lamp is a cylinder.
   (i) Calculate the circumference of the cylinder. Take \( \pi \) to be 3.14 or use the \( \pi \) key on your calculator.
   (ii) Calculate the volume of the cylinder.

17. The diagram represents a tea packet in the shape of a cuboid.

   (a) Calculate the volume of the packet. There are 125 grams of tea in a full packet.
Jason has to design a new packet that will contain 100 grams of tea when it is full.

(b) (i) Work out the volume of the new packet.
(ii) Express the weight of the new tea packet as a percentage of the weight of the packet shown.

The new packet of tea is in the shape of a cuboid. The base of the new packet measures 7 cm by 6 cm.

(c) (i) Work out the area of the base of the new packet.
(ii) Calculate the height of the new packet.
**Plans and Elevations**

The *plan* of a solid is the view from *above*.

The *elevation* of a solid is the view from a *side*. We often use the terms *front elevation* and *side elevation*.

**Task 1.1-21**

Draw the plan, front elevation and side elevation of the shed in the diagram.

**Solution.**

To draw the plan and elevation, look at the shed as shown below.
Task 1.1-22

Draw the front elevation, side elevation and plan of this shape.

Solution.

Looking from the front gives the

*Front elevation*

Looking from the side gives the

*Side elevation*

Looking from above gives the

*Plan*
**Exercises**

1. Draw the front elevation, plan and side elevation for each solid shown below.

2. Draw the plan and front elevation of a square based pyramid that has a height of 6 cm and base with sides of 5 cm.

3. Draw a plan and front elevation for:
   a. Tin of baked beans
   b. A letter box
   c. A roll of sellotape
   d. A ball

4. A pencil with hexagonal cross section stands on one end with its point up. Draw a plan, front elevation and side elevation of the pencil.

5. Draw the front elevation, side elevation and plan of the solids below.
Using Isometric Paper

The spots on isometric paper are arranged at the corners of equilateral triangles.

![Isometric Paper](image)

Task 1.1.23

Draw a cube with sides of 2 cm on isometric paper.

**Solution.**

The diagrams show the three stages needed to draw the cube.

![First draw the base.](image)

![Then add the upright edges.](image)

![Finally add the top of the cube.](image)

Task 1.1.24

The diagrams below show the plan and elevations of a solid object.

![Plan](image) ![Front elevation](image) ![Side elevation](image)
Draw the object in isometric paper.

**Solution.**

From the front elevation the front can be drawn.

Then using the side view elevation the lower part of the side can be drawn.

Finally, the drawing can be completed.
Exercises

1. The diagram shows part of the drawing of a cuboid. Copy and complete the cuboid.

2. On isometric paper draw cubes with sides of lengths 1 cm and 3 cm.

3. A cuboid has sides of length 3 cm, 4 cm and 5 cm. Draw the cuboid in three different ways on isometric paper.

4. In each case below the plan and two elevations of a solid are given. Draw an isometric drawing of each solid.
5. Four cubes with sides of length 1 cm can be joined together in different ways. One way is shown below.
Find other ways in which the four cubes could be joined together.

6. Complete the drawing of a cuboid. One edge is drawn for you.

7. The diagram below is the net of a small open box, with no top face.

   ![Diagram of a net of a small open box]

   a. Find the perimeter of the net
   b. Calculate the area of the net
   c. Copy the diagram, and add one more rectangle in a suitable position to change the diagram to the net of a closed box.
   d. Write down the length, width and height of the box (in any order).
   e. Calculate the volume of the box.
   f. Draw an isometric view of the closed box on a grid like the one below.
Discrete and Continuous Measures

Quantities are said to be *discrete* if they can only take particular values.

For example,

- Number of trainees in a class: 28, 29, 30, 31, 32...
- Shoe size: 6 ½, 7, 7 ½, 8, 8 1/2, 9 ...
- Bottle of milk in a fridge: 0, 1, 2, 3, ...

Quantities that can take any value within a range are said to be *continuous*.

For example, *height, weight, Time*

**Task 1.1-25**

Which of the following are *discrete* and which are *continuous*? For those that are discrete give an example of an impossible value.

(a) Temperature in a classroom.
(b) Votes cast in an election.
(c) Number of cars parked in a car park.
(d) Length of a piece of paper.

**Solution.**

(a) The temperature in a classroom can take any value, and so this is continuous.

(b) Only single votes can be cast in an election and so the number is discrete.

For example, it would be impossible to have 32\(\frac{1}{2}\) votes.

(c) Only complete cars can be parked and so this is discrete. It is impossible to have 22\(\frac{1}{2}\) cars parked in a car park.

(d) The length of a piece of paper is a continuous quantity as it can take any value.

**Task 1.1-26**

(a) The length of a piece of rope is given as 21.4 m. What range of possible values could the length of rope lie within?
(b) The number of cars parked in a car park is said to be 43. Is this number exact or does it represent a range?

**Solution.**

a. As the length of the rope is a continuous quantity it could take any value. If it is quoted as being 21.4 m it must lie in the range
\[21.35 \leq \text{length} < 21.45\]
b. As the number of cars is a discrete quantity, the number is exact and does not represent a range.
Task 1.1-27

1. State whether each of the following is discrete or continuous
   a. Volume of water in a glass
   b. Number of fish in a tank
   c. The population of France
   d. The length of a phone call
   e. The lengths of plants
   f. The number of words in an essay
   g. The time spent on homework
   h. The number of computers in a school
   i. The time it takes to get to school
   j. The weight of a cake
   k. The number of pupils in a school
   l. The distance pupils travel to school

2. Give an example of a continuous quantity and a discrete quantity

3. In each case state whether the value given is exact or give the range of values in which it could lie.
   a. Distance school is 4.63 miles.
   b. Shirt size is 12 ½
   c. Weight of an apple is 125 grams.
   d. Height is 162 cm.
   e. Number of pages in a book is 264.
   f. Volume of drink in a glass is 52.2 cm²
   g. 22.2 cm of rain fell in a mouth
   h. The attendance at a football was 24,731.
   i. The weight of a letter was 54 grams.
   j. A total of £42.63 was raised at a cake stall.
Areas of Parallelograms, Trapeziums, Kites and Rhombuses

The formulae for calculating the areas of these shapes are:

**Parallelogram**

\[ A = bh \]

**Trapezium**

\[ A = \frac{1}{2}(a + b)h \]

**Kite**

\[ A = \frac{1}{2}ab \]

The area of a rhombus can be found using either the formula for a kite or the formula for a parallelogram.

**Task 1.1-28**

Find the area of this kite.

**Solution.**

Using the formula \( A = \frac{1}{2}ab \)

with \( a = 5 \) and \( b = 8 \) gives

\[ A = \frac{1}{2} \times 5 \times 8 \]

\[ = 20 \text{ cm}^2 \]

**Task 1.1-29**

Find the area of this shape.
Solution.

The shape is made up of a parallelogram and a trapezium.

Area of parallelogram = $2 \times 4$

= $8 \text{ cm}^2$

Area of trapezium $= \frac{1}{2}(8 + 12) \times 3$

= $30 \text{ cm}^2$

Total area $= 8 + 30$

= $38 \text{ cm}^2$
Exercises.

1. Find the area of the following shape.

(a) \[ \text{rectangle with length 4 m and width 3 m} \]

(b) \[ \text{rectangle with length 2.5 m and width 2.2 m} \]

(c) \[ \text{rectangle with length 8 cm and width 10 cm} \]

(d) \[ \text{rectangle with length 15 cm and width 5 cm} \]

(e) \[ \text{trapezoid with bases 6 cm and 12 cm, and height 8 cm} \]

(f) \[ \text{rectangle with length 12 m and width 10 m} \]

(g) \[ \text{rectangle with length 9 cm and width 5 cm} \]

(h) \[ \text{rhombus with length 3 m and height 4.8 m} \]

(i) \[ \text{polygon with sides 6 cm, 6 cm, 3.4 cm, 3.4 cm} \]

2. The diagram shown the end wall of a wooden garden shed, (a) Find the area of this end of the shed.

The other end of the shed is identical. The sides are made up of two rectangles of length 3 m.
(b) Find the area of each side of the shed.
(c) Find the total area of the walls of the shed.

3. The diagrams shows the vertical side of a swimming pool.

(a) Find the area of the side of the pool.
(b) Find the area of the rectangular end of the pool.
(c) Find the area of the horizontal base of the pool.
(d) Find the total area of the sides and horizontal base of the pool.

4. Is a car park, spaces are marked out in parallelograms.

Find the area of each parking space.

5. The diagram shows a window of a car:

Find the area of the window

6. A kite is cut out of a sheet of plastic as shown.
   a) Find the area of the kite.
   b) Find the area of the plastic that would be wasted
   c) Would you obtain similar results if you cut a kite
      out of a rectangle of plastic with dimensions 140
      cm by 80 cm?
7. Find the area of each of the following shape.

8. A simple picture frame is made by joining four trapezium shaped strips of wood. Find the area of each trapezium and the total area of the frame.

9. Four rods are joined together to form a parallelogram.
a. Find the area of the parallelogram if:
   i. $h = 2\text{ cm}$
   ii. $h = 4\text{ cm}$
   iii. $h = 5\text{ cm}$

b. Can $h$ be higher than 6 cm?

c. What is the maximum possible area of the parallelogram?

10. 
   a. Find the area of parallelogram ABCD.
   b. Find the area of the triangle ABC.

11. 

What is the area of the kite ABCD equal to twice the area of the triangle ABD?
Surface Area

The net of a cube can be used to find its surface area.

The net is made up of 6 squares, so the surface area will be 6 times the area of one square. If \( x \) is the length of the sides of the cube its surface area will be \( 6 \times x^2 \).

This diagram shows the net for a cuboid. To find the surface area the area of each of the 6 rectangles must be found and then added to give the total.

If \( x \), \( y \) and \( z \) are the lengths of the sides of the cuboid, then the area of the rectangles in the net are as shown here.

\[ A = 2xy + 2yz + 2xz \]

To find the surface area of a cylinder, consider how a cylinder can be broken up into three parts, the top, bottom and curved surface.
The areas of the top and bottom are the same and each is given by $\pi r^2$. The curved surface is a rectangle. The length of one side is the same as the circumference of the circles, $2\pi r$, and the other side is simply the height of the cylinder, $h$. So the area is $2\pi rh$.

The total surface area of the cylinder is

$$2\pi r^2 + 2\pi rh$$

**Task 1.1-28**

Find the surface area of the cuboid shown in the diagram.

**Solution.**

The diagram shows the net of the cuboid and the areas of the rectangles that it contains.

Using the net, the total surface area is given by

$$A = 2 \times 20 + 2 \times 30 + 2 \times 24$$

$$= 148 \text{ cm}^2$$
**Task 1.1-29**

Cans are made out of aluminum sheets, and are cylinders of radius 3 cm and height 10 cm. Find the area of aluminum needed to make one can.

**Solution.**

The diagram shows the two circles and the rectangle from which cans will be made.

The rectangle has one side as 10 cm, the height of the cylinder and the other side is $2 \times \pi \times 3$ cm, the circumference of the top and bottom.

The area of the rectangle is $10 \times 2 \times \pi \times 3$

The area of each circle is $\pi \times 3^2$

So the total surface area is $A = 10 \times 2 \times \pi \times 3 + 2 \times \pi \times 3^2$

$= 245.04$ cm$^2$ (to 2 d.p.)
Exercises.

1. Find the surface area of each of the following cubes or cuboids.

   (a) [Diagram of a cube with sides 4 cm]
   (b) [Diagram of a cuboid with dimensions 7 cm x 2 cm x 6 cm]
   (c) [Diagram of a cube with sides 5 cm x 6 cm x 8 cm]
   (d) [Diagram of a cuboid with dimensions 5 cm x 2 cm x 3 cm]
   (e) [Diagram of a cuboid with dimensions 5 cm x 10 cm x 5 cm]
   (f) [Diagram of a cuboid with dimensions 14 m x 1.2 m x 5.2 m]

2. Find the total surface area of each cylinder shown below.

   (a) [Diagram of a cylinder with height 10 cm and radius 1.5 cm]
   (b) [Diagram of a cylinder with height 6 cm and radius 12 cm]
   (c) [Diagram of a cylinder with height 10 cm and radius 8 cm]
   (d) [Diagram of a cylinder with height 10 m and radius 2 m]
3. Show that each of the cylinders below has the same surface area and find which has the biggest volume.

4. Show that each of the three cuboids below has the same volume. Which has the smallest surface area?

5. A gardener uses a roller to flatten the grass on a lawn. The roller consists of a cylinder of radius 30 cm and width 70 cm.
   (a) Find the area of grass that the roller covers as the cylinder completes 1 rotation.
   (b) If the roller is pulled 5 m, what area of grass does the roller flatten?

6. The volume of a cube is 343 cm$^3$. Find the surface area of the cube.
7. The surface area of a cube is 150 cm$^2$. Find the volume of the cube.
8. A matchbox consists of a tray that slides into a sleeve. If the tray and sleeve have the same dimensions and no material is used up in joins, find:
(a) the area of card needed to make the tray,
(b) the area of card needed to make the sleeve,
(c) the total area of the card needed to make the matchbox.

9. Draw a net of the prism shown in the diagram and use it to find the surface area of the prism.

10. A car tire can be thought of as a hollow cylinder with a hole cut out of the center.

Find the surface area of the outside of the tire.
The diagram shows a cuboid.
The co-ordinates of P are (3, 4, 0).
The co-ordinates of Q are (3, 9, 0).
The co-ordinates of C are (–1, 9, 6).
(a) Write down the (x y z) co-ordinates
    (i) of R,  
    (ii) of B.

(b) Write down the lengths of each of the following edges of the cuboid.
    (i) PQ,  
    (ii) QR.

(c) Calculate the total surface area of the cuboid.
Mass, Volume and Density

*Density* is a term used to describe the mass of a unit of volume of a substance. For example, if the density of a metal is 2000 kg/m$^3$, then 1 m$^3$ of the substance has a mass of 2000 kg.

*Mass, volume and density* are related by the following equations.

\[
\text{Mass} = \text{Volume} \times \text{Density}
\]
\[
\text{Volume} = \frac{\text{Mass}}{\text{Density}}
\]

or

\[
\text{Density} = \frac{\text{Mass}}{\text{Volume}}
\]

The density of water is 1 gram/cm$^3$ (or 1 g cm$^{-3}$) or 1000 kg/m$^3$ (or 1000 kg m$^{-3}$).

**Task 1.1-30**

Find the mass of water in the fish tank shown in the diagram.

![Diagram of a fish tank](image)

**Solution**

First calculate the volume of water.

\[ V = 25 \times 30 \times 20 \]
\[ = 15000 \text{ cm}^3 \]

Now use \( \text{Mass} = \text{Volume} \times \text{Density} \)

\[ = 15000 \times 1 \text{ (as density of water } = 1\text{g/cm}^3) \]
\[ = 15000 \text{ grams} \]
\[ = 1 \text{ kg} \]
Task 1.1-31

The block of metal shown has a mass of 500 grams.

Find its density in

(a) \( \text{g/cm}^3 \)
(b) \( \text{kg/m}^3 \)

**Solution**

(a) First find the volume.

\[
\text{Volume} = 5 \times 8 \times 10 = 400 \, \text{cm}^3
\]

Then use

\[
\text{Density} = \frac{\text{Mass}}{\text{Volume}}
\]

\[
\text{Density} = \frac{500}{400} = 1.25 \, \text{g/cm}^3
\]

(b) The process can then be repeated working in kg and m.

\[
\text{Volume} = 0.05 \times 0.08 \times 0.1 = 0.0004 \, \text{m}^3
\]

\[
\text{Density} = \frac{0.5}{0.0004} = \frac{5000}{4} = 1250 \, \text{kg/m}^3
\]
Exercises

1. A drinks carton is a cuboid with size as shown.
   (a) Find the volume of the carton.
   (b) If it contains 8 cm³ of air, find the volume of the drink.
   (c) Find the mass of the drink if it has a density of 1 gram / cm³.

2. The diagram shows a concrete block of mass 6 kg.
   (a) Find the volume of the block.
   (b) What is the density of the concrete?

3. A ream (500 sheets) of paper is shown in the diagram.
   If the mass of the ream is 2.5 kg, find the density of the paper.

4. A barrel is a cylinder with radius 40 cm and height 80 cm. It is full of water.
   (a) Find the volume of the barrel.
   (b) Find the mass of the water in the barrel.

5. A metal bar has a cross section with an area of 3 cm² and a length of 40 cm. Its mass is 300 grams.
   (a) Find the volume of the bar.
   (b) Find the density of the bar.
   (c) Find the mass of another bar with the same cross section and length 50 cm.
   (d) Find the mass of a bar made from the same material, but with a cross section of area 5 cm² and length 80 cm.

6. A bottle which holds 450 cm³ of water has a mass of 530 grams. What is the mass of the empty bottle?

7. The diagram shows the dimensions of a swimming pool.
   (a) Find the volume of the swimming pool.
   (b) Find the mass of water in the pool if it is completely full.
   (c) In practice, the level of the water is 20 cm below the top of the pool.
       Find the volume and mass of the water in this case.
8. The density of a metal is 3 grams / cm$^3$. It is used to make a pipe with external radius of 1.5 cm and an internal radius of 0.5 cm.
   (a) Find the area of the cross section of the pipe.
   (b) If the length of the pipe is 75 cm, find its mass.
   (c) Find the length, to the nearest cm, of a pipe that has a mass of 750 grams.

9. A foam ball has a mass of 200 grams and a radius of 10 cm.
   (a) Use the formula $V = \frac{4\pi r^3}{3}$ to find the volume of the ball.
   (b) Find the density of the ball.
   (c) The same type of foam is used to make a cube with sides of length 12 cm. Find the mass of the cube.

10. The diagram shows the cross section of a metal beam. A 2 m length of the beam has a mass of 48 kg.
    (a) Find the density of the metal.
    A second type of beam uses the same type of metal, but all the dimensions of the cross section are increased by 50%.
    (b) Find the length of a beam of this type that has the same mass as the first beam.
Volumes, Areas and Lengths

The area of a triangle with side lengths \(a\) and \(b\) and included angle \(\theta\) is given by \(\frac{1}{2}ab \sin \theta\).

\[
A = \frac{1}{2}ab \sin \theta
\]

Proof

If you construct the perpendicular from the vertex \(B\) to \(AC\), then its length, \(p\), is given by

\[
p = a \sin \theta
\]

Thus the area of \(ABC\) is given by

\[
\text{area} = \frac{1}{2} \times \text{base} \times \text{height}
\]

\[
= \frac{1}{2} \times b \times p
\]

\[
= \frac{1}{2} \times b \times (a \sin \theta)
\]

\[
= \frac{1}{2}ab \sin \theta
\]

as required.

The volumes of a pyramid, a cone and a sphere are found using the following formulae.

**Pyramid**

\[
V = \frac{1}{3}Ah
\]

**Cone**

\[
V = \frac{1}{3}\pi r^2h
\]

**Sphere**

\[
V = \frac{4}{3}\pi r^3
\]

The proofs of these results are rather more complex and require mathematical analysis beyond the scope of this text.
A part of the circumference of a circle is called an arc. If the angle subtended by the arc at the centre of the circle is \( \theta \) then the arc length \( l \) is given by
\[
l = \frac{\theta}{360^\circ} \times 2\pi r
\]

The region between the two radii and the arc is called a sector of the circle. The area of the sector of the circle is
\[
A = \frac{\theta}{360^\circ} \times \pi r^2
\]

The region between the chord AB and the arc APB is called a segment of the circle. The area of the segment of the circle is
\[
A = \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin \theta
\]

**Task 1.1-32**

The shaded area shows a segment of a circle of radius 64 cm.

The length of the chord AB is 100 cm.

(a) Find the angle \( \theta \), to 2 d.p.

(b) Find the area of triangle OAB.

(c) Find the area of the sector of the circle with angle 2\( \theta \).

(d) Find the area of the segment shaded in the figure.
Glossary

**AND Gate**: a logic circuit designed to compare TRUE-FALSE (or on-off or one-zero) inputs and pass a resultant TRUE signal only when all the inputs are TRUE.

**Binary**: having two components or possible states.

**Binary code**: a system for representing things by combinations of two symbols, such as one and zero, TRUE and FALSE, or the presence or absence of voltage.

**Binary number system**: a number system that uses two as its base and expresses numbers as strings of zeros and ones.

**Bit**: the smallest unit of information in a computer, equivalent to a single zero or one. The word “bit” is a contraction of binary digit.

**Boolean algebra**: a method for expressing the logical relationships between entities such as propositions or on-off computer circuit; invented by the 19th Century English mathematician George Boole.

**Byte**: a sequence of bits, usually eight, treated as a unit for computation or storage.

**Carry**: the digit added to the next column in a an additional problem when the sum of the numbers in a column equals or exceeds the number base.

**Kilobyte**: 1,024 bytes (1.024 being one K, or two to the 10th power); often used as a measure of memory capacity.

**Logic Gate**: a circuit that accepts one or more than one input and always produces a single predictable output.

**OR gate**: a circuit designed to compare binary TRUE-FALSE (or on-off or one-zero) inputs and pass a resultant TRUE signal if any input is TRUE.

**XOR gate**: a circuit designed to compare binary TRUE-FALSE (or on-off or one-zero) inputs and pass a resultant TRUE signal if a single input is TRUE, otherwise the output is FALSE.

1 Bit = Binary digit
1 Byte = 8 Bits
1 Kilobyte = 1,000 Bytes
1 Megabyte = 1,000 Kilobytes
1 Gigabyte = 1,000 Megabytes
1 Terabyte = 1,000 Gigabyte
1 Petabyte = 1,000 Terabytes
1 Exabyte = 1,000 Petabytes
1 Zettabyte = 1,000 Exabytes
1 Yottabyte = 1,000 Zettabytes

The Binary System

History
The concept of using two symbols to encode information is an old one. African bush tribes sent messages via a combination of high and low pitches. Australian aborigines and New Guinea Tribesman counted by two's. Even more recently, Morse code consists of groups of dots and dashes which represents letters of the alphabet in another two-symbol code.

Timeline

In 1666, Gottfried Wilhelm Leibniz wrote the essay "De Arte Combinatoria" which laid a method for expressing all things in law of thought which consists of precision mathematics.

In the 19th Century, British mathematician George Boole invents the system of symbolic logic call Boolean algebra.
In 1867, Charles Sanders Peirce introduces Boolean algebra to the United States.

In 1936 Claude Shannon bridges the gap between algebraic theory and practical application.

### Binary To Decimal Conversion

<table>
<thead>
<tr>
<th>Binary Evaluate</th>
<th>$2^4$</th>
<th>$2^3$</th>
<th>$2^2$</th>
<th>$2^1$</th>
<th>$2^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal Value</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Value</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

### Decimal To Binary Conversion

To convert a decimal number to binary, first subtract the largest possible power of two, and keep subtracting the next largest possible power from the remainder, marking 1s in each column where this is possible and 0s where it is not.

**Example 1 - (Convert Decimal 44 to Binary)**

To convert the decimal number 44 to binary:

1. 16 (is 16, place a 1 in $2^4$ column)
2. 28 (44 - 16 = 28, place a 1 in $2^4$ column)
3. 10 (28 - 16 = 12, place a 1 in $2^4$ column)
4. 8 (12 - 8 = 4, place a 1 in $2^4$ column)
5. 0 (4 - 4 = 0, place a 1 in $2^4$ column)

Therefore, the binary representation of 44 is 101100.
Example 2 - (Convert Decimal 15 to Binary)

\[
\begin{array}{cccccc}
4 & 4 \\
- & 3 & 2 \\
1 & 2 \\
- & 8 \\
\hline
4 \\
- & 4 & 32 & 16 & 8 & 4 & 2 & 1 \\
\hline
0 & 1 & 0 & 1 & 1 & 0 & 0
\end{array}
\]

Example 3 - (Convert Decimal 62 to Binary)

\[
\begin{array}{cccccc}
6 & 2 \\
- & 3 & 2 \\
3 & 0 \\
- & 1 & 6 \\
\hline
1 & 4 \\
- & 8 \\
\hline
6 \\
- & 4 \\
\hline
2 & 32 & 16 & 8 & 4 & 2 & 1 \\
- & 2 & 1 & 1 & 1 & 1 & 0
\end{array}
\]

Binary Addition

Example 1
Example 2

Base 2 Place  \[ 2^3 \ 2^2 \ 2^1 \ 2^0 \]
Carryover  
\[
\begin{array}{c}
1 \\
+ 1 \\
1 1 0
\end{array} + 3
\]
\[
\begin{array}{c}
1 \\
1 1 0
\end{array} + 6
\]

Example 3

Base 2 Place  \[ 2^3 \ 2^2 \ 2^1 \ 2^0 \]
Carryover  
\[
\begin{array}{c}
1 \\
+ 1 \\
1 1 1
\end{array} + 3
\]
\[
\begin{array}{c}
1 \\
1 1 0
\end{array} + 14
\]

Example 4

Base 2 Place  \[ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \]
Carryover  
\[
\begin{array}{c}
1 \\
+ 1 \\
1 0 1
\end{array} + 2
\]
\[
\begin{array}{c}
1 \\
1 0 1
\end{array} + 10
\]

Example 5
Binary Multiplication

Example 1

Base 2 Place  \(2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0\)

\[
\begin{array}{c}
1 \\
1 \\
1 \\
\end{array}
\xrightarrow{x}
\begin{array}{c}
1 \\
1 \\
1 \\
\end{array}
\]

1 1 1 1

\[
\begin{array}{c}
1 \\
1 \\
1 \\
\end{array}
\xrightarrow{\text{Carry}}
\begin{array}{c}
1 \\
0 \\
0 \\
1 \\
\end{array}
\]

Example 2

Base 2 Place  \(2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0\)

\[
\begin{array}{c}
1 \\
0 \\
0 \\
1 \\
\end{array}
\xrightarrow{x}
\begin{array}{c}
1 \\
0 \\
\end{array}
\]

0 0 0 0

\[
\begin{array}{c}
0 \\
0 \\
1 \\
1 \\
\end{array}
\xrightarrow{\text{Carry}}
\begin{array}{c}
1 \\
0 \\
0 \\
1 \\
\end{array}
\]

Example 3

Base 2 Place  \(2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0\)

\[
\begin{array}{c}
1 \\
0 \\
1 \\
1 \\
\end{array}
\xrightarrow{x}
\begin{array}{c}
1 \\
1 \\
\end{array}
\]

1 1 0 1 1

\[
\begin{array}{c}
1 \\
0 \\
1 \\
\end{array}
\xrightarrow{\text{Carry}}
\begin{array}{c}
1 \\
0 \\
0 \\
0 \\
1 \\
\end{array}
\]

1 0 0 0 0 1
ACKNOWLEDGEMENT

This draft was prepared at the Competency-Based Learning Materials Writeshop conducted at the Agusan del Sur School of Arts and Trades.

It was based on the Core Competency of the Training Regulation on Computer Hardware Servicing NC II

Some materials for contextual learning were supplied by both the Academic and Vocational Instructors. Some readings and references were also provided by the references of the previous learning’s, trainings and seminars.

This learning instrument was developed by the following personnel:

Technology Instructor:
Lonie Rex T. Escobar – Instructor III
Nicolas H. Deroca – Asst. Professor IV – VIS Designate